

# Behaviour of a spin-1/2 particle in Schwarzschild embedded in an electromagnetic universe

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## Abstract

The Dirac equation is considered in Schwarzschild black hole immersed in an electromagnetic universe with charge coupling. The equations of the charged spin-1/2 particle is separated into radial and angular equations by adopting the Newman-Penrose formalism. The angular equations obtained are similar to the Schwarzschild geometry. For the radial equations we manage to obtain the one dimensional Schrödinger-type wave equations with effective potentials. Due to the presence of electromagnetic field from the surroundings, the interaction with the charged spin-1/2 is considered. Finally, we study the behaviour of the potentials by plotting them as a function of radial distance and expose the effect of the external parameter, charge and the frequency of the particle on them.

## 1 Introduction

In general relativity, Schwarzschild (S) black hole is the solution to the Einstein field equations that describes the gravitational field outside a spherical mass, while Reissner-Nrdström (RN) black hole is the solution to the Einstein-Maxwell field equations that describes the gravitational field outside a spherical charged mass. Thus the electromagnetic (em) field in RN solution originates from the S mass. The above well known solutions S and RN, are isolated system. However, the notion of considering the system in a uniform external em field is a well known subject by now. For example the S black hole immersed in a homogeneous em field has been introduced in [1-3]. This represents an external field of non rotating uncharged mass immersed in external em and gravitational field, Bertotti-Robinson (BR) universe[4-5]. Mathematically, they interpolate two exact solutions of Einstein's equations. As a result of this embedding the horizon and particle geodesics are modified. The extension of an S source in an external em field has been obtained by Halilsoy et. al. [6]. In the last reference, they present the metric of a S black hole coupled to an external, stationary em BR solution. Limiting cases of their metric include the case of a stationary em universe in which conformal curvature arises due to rotation. Again, the horizon radius of such a black hole and particle geodesics changed. Later on, the same authors in [6] have introduced a new metric describes the RN black hole coupled to an external, stationary em static field [7]. We will in this paper study the Dirac equation by using the metric obtained in [7].

On the other hand, the behaviour of a spin-1/2 particle in different backgrounds has been studied by many authors [8-10]. Similarly, the behaviour of

a spin-1/2 particle around a charged black hole is studied by Mukhopadhyay [11], where he studied the behaviour of the potential by varying the charge of the black hole. He also study the space-dependent reflection and transmission coefficients and showed that as the potential barrier level decreases, the corresponding transmission probability increases. The separation of variables of the spin-1 and 3/2 field equation is performed in details in the S geometry by means of the Newman Penrose (NP) formalism [12,13]. Zecca showed that, as a consequence of the particular nature of the spin coefficients it is shown, by induction, that the massive field equations can be separated for arbitrary spin. Recently, Dirac equation was examined in Kerr-Taub-NUT spacetime [14], by using Boyer-Lindquist coordinates they obtained exact solution of the angular equations for some special cases and exposed the effect of the NUT parameter from the effective potentials plots. Other studies of Dirac equation in different backgrounds, Nutku helicoid spacetime [15], Kerr–Newman–AdS black hole geometry [16] and in the background of the Kerr–Newman family, which has been considered in several studies [17-18].

The spacetime we are considering consists of a central mass plus an external em radiation which may not be attributed to the charge of the mass [1,7]. The metric that describe this spacetime interpolates the two well-known solutions of general relativity, the S and the BR solutions. We shall refer to this solution as the S em black hole (SEBH). Here, we study the solution of the Dirac equation in SEBH. The set of equations representing the Dirac equation in the NP formalism is decoupled into a radial (function of distance  $r$  only) and an angular parts (function of angle  $\theta$  only). The solution of the angular equation is given in terms of standard spherical harmonics as in the S geometry. The radial equations are discussed and the radial wave equations with effective potentials are obtained. Finally in order to understand and expose the effect of the external parameter, charge and the frequency of the spin-1/2 particle on the potentials, curves are plotted and discussed.

Our paper is organized as follows: in section 2, we present the Dirac equation in SEBH and separate them into two parts. In section 3, solutions of the angular and radial equations. In section 4, we study the behaviour of the potentials by varying the external parameter, the charge and the frequency by plotting the potentials as a function of radial distance. Finally we make concluding remarks and digress about future applications of this research.

## 2 Dirac equation in SEBH

The SEBH metric we are dealing with represents the non-linear superposition of the S solution and the BR solution[1,7], is given by

$$ds^2 = \frac{\Delta}{r^2} dt^2 - \frac{r^2}{\Delta} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where

$$\Delta = r^2 - 2Mr + M^2 (1 - a^2). \quad (2)$$

in which,  $M$  is the S mass coupled to an external electromagnetic field and  $a$  ( $0 < a \leq 1$ ) is the external parameter. This metric satisfies all the required limits as boundary conditions:

$$\begin{aligned} (a = 1) &\rightarrow (the\ S\ Solution) \\ (0 < a \leq 1) &\rightarrow (the\ S + BR\ or\ SEBH\ Solution). \end{aligned} \quad (3)$$

The case ( $a = 0$ ), which is excluded, is the extremal RN which is transformable to BR metric. As a result of the coupling of S and BR, the horizon shrinks to a significant degree. The horizon is given by

$$r_h = M(1 + a). \quad (4)$$

It is clear that  $r_h \leq 2m$ , since ( $0 < a \leq 1$ ). The vector potential for the above metric is given by

$$A_\mu = \left( \frac{M\sqrt{1-a^2}}{2a(r-r_h)}, 0, 0, 0 \right). \quad (5)$$

Using NP formalism, the Dirac equation can be written as

$$\sigma_{AB'}^i \partial_i P^A + i\mu_p \bar{Q}_{B'} \epsilon_{C'B'} = 0 \quad (6a)$$

$$\sigma_{AB'}^i \partial_i Q^A + i\mu_p \bar{P}_{B'} \epsilon_{C'B'} = 0 \quad (6b)$$

where  $P^A$  and  $Q^A$  are the pair of spinors.  $\mu_p/\sqrt{2}$  is the mass of the particles and  $\sigma_{AB'}^i$  is the Pauli matrix.

Introducing the null tetrad 1-form of NP formalism, we make the choice of the following null tetrad basis 1-forms ( $l, n, m, \bar{m}$ ) of the NP formalism [19, 20] in terms of this new basis Pauli matrices can be written as

$$\sigma_{AB'}^i = \frac{1}{\sqrt{2}} \begin{pmatrix} l^\mu & m^\mu \\ \bar{m}^\mu & n^\mu \end{pmatrix} \quad (7)$$

Now considering  $B' = 0$ , and subsequently  $B' = 1$ , in Eq. (6a) we obtain

$$l^\mu (\partial_\mu + iqA_\mu) P^0 + \bar{m}^\mu (\partial_\mu + iqA_\mu) P^1 + (\Gamma_{1000'} - \Gamma_{0010'}) P^0 \quad (8)$$

$$+ (\Gamma_{1100'} - \Gamma_{0110'}) P^1 - i\mu_0 \bar{Q}^{1'} = 0,$$

$$m^\mu (\partial_\mu + iqA_\mu) P^0 + n^\mu (\partial_\mu + iqA_\mu) P^1 + (\Gamma_{1001'} - \Gamma_{0011'}) P^0 \quad (9)$$

$$+ (\Gamma_{1101'} - \Gamma_{0111'}) P^1 + i\mu_0 \bar{Q}^{0'} = 0,$$

where  $\mu_0 = \sqrt{2}\mu_p$  and  $q$  are the mass and the charge of the Dirac particle respectively.  $A_\mu$  is the electromagnetic vector potential. Now by taking the complex conjugation of Eq. (6b) and writing various spin coefficients by their

named symbols as in [19,20] and choosing  $P^0 = F_1, P^1 = F_2, \bar{Q}^{0'} = G_2, \bar{Q}^{1'} = G_1$ , we obtain the Dirac equation in the NP formalism [19,20] as

$$(l^\mu \partial_\mu + iql^\mu A_\mu + \epsilon - \rho) F_1 + (\bar{m}^\mu \partial_\mu + iq\bar{m}^\mu A_\mu + \pi - \alpha) F_2 = i\mu_0 G_1, \quad (10a)$$

$$(n^\mu \partial_\mu + iqn^\mu A_\mu + \mu - \gamma) F_2 + (m^\mu \partial_\mu + iqm^\mu A_\mu + \beta - \tau) F_1 = i\mu_0 G_2, \quad (10b)$$

$$(l^\mu \partial_\mu + iql^\mu A_\mu + \bar{\epsilon} - \bar{\rho}) G_2 - (m^\mu \partial_\mu + iqm^\mu A_\mu + \bar{\pi} - \bar{\alpha}) G_1 = i\mu_0 F_2, \quad (10c)$$

$$(n^\mu \partial_\mu + iqn^\mu A_\mu + \bar{\mu} - \bar{\gamma}) G_1 - (\bar{m}^\mu \partial_\mu + iq\bar{m}^\mu A_\mu + \bar{\beta} - \bar{\tau}) G_2 = i\mu_0 F_1. \quad (10d)$$

We will now study the Dirac equation (10) in the background of metric (1). Let us write the null tetrad basis vectors of the null tetrad as

$$\begin{aligned} l_\mu &= dt - \frac{r^2 dr}{\sqrt{\Delta}}, \\ n_\mu &= \frac{\Delta}{2r^2} dt + \frac{1}{2} dr, \\ m_\mu &= \frac{-r}{\sqrt{2}} (d\theta + i \sin \theta d\phi), \end{aligned} \quad (11)$$

and

$$\begin{aligned} l^\mu &= \frac{r^2}{\Delta} dt + dr, \\ n^\mu &= \frac{1}{2} dt - \frac{\Delta}{2r^2} dr, \\ m^\mu &= \frac{1}{\sqrt{2}r} (d\theta + \frac{i}{\sin \theta} d\phi), \end{aligned} \quad (12)$$

Using the above tetrad we determine the nonzero NP complex spin coefficients [20] as,

$$\begin{aligned} \rho &= -\frac{1}{r}, \quad \mu = \frac{-\Delta}{2r^3}, \\ \gamma &= \frac{r-M}{2r^2} - \frac{\Delta}{2r^3}, \\ \alpha &= -\beta = \frac{-\cot \theta}{2\sqrt{2}r}. \end{aligned} \quad (13)$$

We will consider the corresponding Compton wave of the Dirac particle as in the form of  $F = F(r, \theta) e^{i(kt+m\phi)}$ , where  $k$  is the frequency of the incoming wave and  $m$  is the azimuthal quantum number of the wave. The form of the Dirac equation suggests that we assume [19],

$$\begin{aligned} rF_1 &= f_1(r, \theta) e^{i(kt+m\phi)}, \\ F_2 &= f_2(r, \theta) e^{i(kt+m\phi)}, \end{aligned}$$

$$\begin{aligned} G_1 &= g_1(r, \theta) e^{i(kt+m\phi)}, \\ rG_2 &= g_2(r, \theta) e^{i(kt+m\phi)}. \end{aligned} \quad (14)$$

Substituting the appropriate spin coefficients (13) and the spinors (14) into the Dirac equation (10), we obtain

$$\mathbf{D}f_1 + \frac{1}{\sqrt{2}}\mathbf{L}f_2 = i\mu_0 r g_1, \quad (15a)$$

$$\frac{\Delta}{2}\mathbf{D}^\dagger f_2 - \frac{1}{\sqrt{2}}\mathbf{L}^\dagger f_1 = -i\mu_0 r g_2, \quad (15b)$$

$$\mathbf{D}g_2 - \frac{1}{\sqrt{2}}\mathbf{L}^\dagger g_1 = i\mu_0 r f_2, \quad (15c)$$

$$\frac{\Delta}{2}\mathbf{D}^\dagger g_1 + \frac{1}{\sqrt{2}}\mathbf{L}g_2 = -i\mu_0 r f_1, \quad (15d)$$

where

$$\mathbf{D} = \frac{d}{dr} + i\frac{kr^2}{\Delta} + i\frac{qMr^2\sqrt{1-a^2}}{\Delta 2a(r-r_h)} \quad (16a)$$

$$\mathbf{D}^\dagger = \frac{d}{dr} - i\frac{kr^2}{\Delta} - i\frac{qMr^2\sqrt{1-a^2}}{\Delta 2a(r-r_h)} + \frac{r-M}{\Delta} \quad (16b)$$

$$\mathbf{L} = \frac{d}{d\theta} + \frac{m}{\sin\theta} + \frac{\cot\theta}{2} \quad (16c)$$

$$\mathbf{L}^\dagger = \frac{d}{d\theta} - \frac{m}{\sin\theta} + \frac{\cot\theta}{2} \quad (16d)$$

It is now apparent that Eqs. (15) can be separated by implying the separability ansatz

$$\begin{aligned} f_1 &= R_1(r) A_1(\theta), & f_2 &= R_2(r) A_2(\theta), \\ g_1 &= R_2(r) A_1(\theta), & g_2 &= R_1(r) A_2(\theta). \end{aligned} \quad (17)$$

With this ansatz, Eqs. (15) become

$$A_1 \mathbf{D}R_1 + \frac{1}{\sqrt{2}}R_2 \mathbf{L}A_2 = i\mu_0 r R_2 A_1, \quad (18a)$$

$$\Delta A_2 \mathbf{D}^\dagger R_2 - \sqrt{2}R_1 \mathbf{L}^\dagger A_1 = -2i\mu_0 r R_1 A_2, \quad (18b)$$

$$A_2 \mathbf{D}R_1 - \frac{1}{\sqrt{2}}R_2 \mathbf{L}^\dagger A_1 = i\mu_0 r R_2 A_2, \quad (18c)$$

$$\Delta A_1 \mathbf{D}^\dagger R_2 + \sqrt{2}R_1 \mathbf{L}A_2 = -2i\mu_0 r R_1 A_1. \quad (18d)$$

These equations (18) imply that

$$\mathbf{D}R_1 - i\mu_0 r R_2 = -\lambda_1 R_2, \quad (19a)$$

$$\Delta \mathbf{D}^\dagger R_2 + 2i\mu_0 r R_1 = \lambda_2 R_1, \quad (19b)$$

$$\mathbf{D}R_1 - i\mu_0 r R_2 = \lambda_3 R_2, \quad (19c)$$

$$\Delta \mathbf{D}^\dagger R_2 + 2i\mu_0 r R_1 = -\lambda_4 R_1, \quad (19d)$$

$$\mathbf{L}A_2 = \lambda_1 A_1, \quad \mathbf{L}^\dagger A_1 = \lambda_2 A_2, \quad (19e)$$

$$\mathbf{L}^\dagger A_1 = \lambda_3 A_2, \quad \mathbf{L}A_2 = \lambda_4 A_1, \quad (19f)$$

where  $\lambda_1, \lambda_2, \lambda_3$  and  $\lambda_4$  are four constants of separation. However, let us assume ( $\lambda_4 = \lambda_1 = -\lambda, \lambda_2 = \lambda_3 = \lambda$ ), then we obtain the radial and the angular pair equations

$$\mathbf{D}R_1 = (\lambda + i\mu_0 r) R_2, \quad \Delta \mathbf{D}^\dagger R_2 = (\lambda - 2i\mu_0 r) R_1, \quad (20)$$

$$\mathbf{L}A_2 = -\lambda A_1, \quad \mathbf{L}^\dagger A_1 = \lambda A_2. \quad (21)$$

### 3 Solution of angular and Radial equations

Angular Eqs. (21) can be written as

$$\frac{dA_1}{d\theta} + \left( \frac{\cot \theta}{2} - \frac{m}{\sin \theta} \right) A_1 = -\lambda A_2, \quad (22)$$

$$\frac{dA_2}{d\theta} + \left( \frac{\cot \theta}{2} + \frac{m}{\sin \theta} \right) A_2 = \lambda A_1. \quad (23)$$

which are exactly the same as the angular equation in the S geometry whose solution is given in terms of standard spherical harmonics [21-23] as

$$A_{1,2} = Y_l^n(\theta), \quad (24)$$

with  $\lambda^2 = (n + \frac{1}{2})^2$ .

The radial equations (20) can be rearranged as

$$\sqrt{\Delta} \left( \frac{d}{dr} + i \frac{kr^2}{\Delta} + i \frac{qMr^2 \sqrt{1-a^2/2a}}{\Delta(r-r_h)} \right) R_1 = (\lambda + i\mu_* r) \sqrt{\Delta} R_2, \quad (25)$$

$$\sqrt{\Delta} \left( \frac{d}{dr} - i \frac{kr^2}{\Delta} - i \frac{qMr^2 \sqrt{1-a^2/2a}}{\Delta(r-r_h)} \right) \sqrt{\Delta} R_2 = (\lambda - i\mu_* r) R_1, \quad (26)$$

where  $\mu_*$  is the normalized rest mass of the spin-1/2 particle.

Our task now is to put the radial equations (25) and (26) in the form of one dimensional wave equations. Therefore, to achieve our task we follow the method applied by Chandrasekhar's book [19]. Starts with the transformations

$$P_1 = R_1, \quad P_2 = \sqrt{\Delta} R_2. \quad (27)$$

With these transformation Eqs. (25) and (26) become

$$\frac{dP_1}{dr} + i \frac{k\Omega}{\Delta} P_1 = \frac{1}{\Delta} (i\mu_* r + \lambda) P_2, \quad (28)$$

$$\frac{dP_2}{dr} - i \frac{k\Omega}{\Delta} P_2 = \frac{1}{\Delta} (\lambda - i\mu_* r) P_1, \quad (29)$$

where

$$\Omega = r^2 + \frac{qMr^2\sqrt{1-a^2}}{2a(r-r_h)k} \quad (30)$$

Assume

$$\frac{du}{dr} = \frac{\Omega}{\Delta}. \quad (31)$$

Then, the above equations (28) and (29) in terms of the new independent variable  $u$ , become

$$\frac{dP_1}{du} + ikP_1 = \frac{\sqrt{\Delta}}{\Omega} (i\mu_* r + \lambda) P_2, \quad (32)$$

$$\frac{dP_2}{du} - ikP_2 = \frac{\sqrt{\Delta}}{\Omega} (\lambda - i\mu_* r) P_1. \quad (33)$$

where

$$u = r - \sqrt{C} \tan^{-1} \left( \frac{r}{\sqrt{C}} \right) - \frac{qMr^2\sqrt{1-a^2}}{2ak(2r_h^2 + 2C)} \left[ \frac{2r_h}{\sqrt{C}} \tan^{-1} \left( \frac{r}{\sqrt{C}} \right) - 2 \log(r - r_h) + \log(C + r^2) \right], \quad (34)$$

and  $C = M^2 - a^2 M^2 - 2Mr$ .

Let us apply another transformation, namely the new functions

$$P_1 = \phi_1 \exp \left( \frac{-i}{2} \tan^{-1} \left( \frac{\mu_* r}{\lambda} \right) \right), \quad P_2 = \phi_2 \exp \left( \frac{i}{2} \tan^{-1} \left( \frac{\mu_* r}{\lambda} \right) \right), \quad (35)$$

and next changing the variable  $u$  into  $\hat{r}$  as  $\hat{r} = u - \frac{1}{2k} \tan^{-1} \left( \frac{\mu_* r}{\lambda} \right)$ , then we can write Eqs.(32) and (33) in the alternative forms

$$\frac{d\phi_1}{d\hat{r}} + ik\phi_1 = W\phi_2, \quad (36)$$

$$\frac{d\phi_2}{d\hat{r}} - ik\phi_2 = W\phi_1, \quad (37)$$

where

$$W = \frac{2k\sqrt{\Delta} (\lambda^2 + \mu_*^2 r^2)^{3/2}}{2kr^2 (\lambda^2 + \mu_*^2 r^2) \left( 1 + \frac{qM\sqrt{1-a^2}}{2a(r-r_h)k} \right) + \Delta \lambda \mu_*}. \quad (38)$$

Finally, in order to put the above equations (36) and (37) into one dimensional wave equations, we define

$$2\phi_1 = \psi_1 + \psi_2, \quad 2\phi_2 = \psi_1 - \psi_2. \quad (39)$$

Then Eqs.(36) and (37) become

$$\frac{d\psi_1}{d\hat{r}} - W\psi_1 = -ik\psi_2, \quad (40)$$

$$\frac{d\psi_2}{d\hat{r}} + W\psi_2 = -ik\psi_1. \quad (41)$$

Which can be cast into

$$\frac{d^2\psi_1}{d\hat{r}^2} + k^2\psi_1 = V_+\psi_1, \quad (42)$$

$$\frac{d^2\psi_2}{d\hat{r}^2} + k^2\psi_2 = V_-\psi_2, \quad (43)$$

where the effective potentials can be obtained from

$$V_{\pm} = W^2 \pm \frac{dW}{d\hat{r}}. \quad (44)$$

We calculate the potentials as

$$\begin{aligned} V_{\pm} = & \frac{\Delta B^3}{D^2} \pm \frac{\sqrt{\Delta} B^{3/2}}{D^2} ((r-M)B + 3\Delta r\mu_*^2) \\ & \mp \frac{\Delta^{3/2} B^{5/2}}{D^3} \left[ (2rB + 2r^3\mu_*^2) \left( 1 + \frac{qM\sqrt{1-a^2}}{2a(r-r_h)k} \right) - r^2 B \frac{qM\sqrt{1-a^2}}{2a(r-r_h)^2 k} + \frac{(r-M)\lambda\mu_*}{k} \right], \end{aligned} \quad (45)$$

where

$$B = (\lambda^2 + \mu_*^2 r^2), \quad D = r^2 B \left( 1 + \frac{qM\sqrt{1-a^2}}{2a(r-r_h)k} \right) + \frac{\Delta\lambda\mu_*}{2k}. \quad (46)$$

Let us note that for the case of  $a = 1$ , the potentials reduced to the the S geometry, namely

$$\begin{aligned} V_{\pm} = & \frac{\Delta L^3}{\left( r^2 B + \frac{\Delta\lambda\mu_*}{2k} \right)^2} \pm \frac{\sqrt{\Delta} B^{3/2}}{\left( r^2 B + \frac{\Delta\lambda\mu_*}{2k} \right)^2} ((r-M)B + 3\Delta r\mu_*^2) \\ & \mp \frac{\Delta^{3/2} B^{5/2}}{\left( r^2 B + \frac{\Delta\lambda\mu_*}{2k} \right)^3} \left[ 2rB + 2r^3\mu_*^2 + \frac{(r-M)\lambda\mu_*}{k} \right]. \end{aligned} \quad (47)$$

Here we are going to find the complete solution of Eqs. (42) and (43). First we can rewrite the equations as

$$\frac{d^2\psi_1}{d\hat{r}^2} + (k^2 - V_+)\psi_1 = 0, \quad (48)$$



$$\frac{d^2 \psi_2}{d\hat{r}^2} + (k^2 - V_-) \psi_2 = 0. \quad (49)$$

This is simply the one dimensional Schrödinger wave equations with potentials  $V_{\pm}$  and the total energy of the wave  $k^2$ . We can solve Eqs. (48) and (49) by WKB approximation method [24,25]. The solution is given by

$$\begin{aligned} \psi_1 &= \sqrt{T_1 [\omega_1 (\hat{r})]} e^{iy_1} + \sqrt{R_1 [\omega_1 (\hat{r})]} e^{-iy_1}, \\ \psi_2 &= \sqrt{T_2 [\omega_2 (\hat{r})]} e^{iy_2} + \sqrt{R_2 [\omega_2 (\hat{r})]} e^{-iy_2}, \end{aligned} \quad (50)$$

where

$$\omega_1 (\hat{r}) = \sqrt{(k^2 - V_+)}, \quad \omega_2 (\hat{r}) = \sqrt{(k^2 - V_-)} \quad (51)$$

$$y_1 (\hat{r}) = \int \omega_1 (\hat{r}) d\hat{r} + \text{constant}, \quad y_2 (\hat{r}) = \int \omega_2 (\hat{r}) d\hat{r} + \text{constant} \quad (52)$$

with

$$T_1 (r) + R_1 (r) = 1, \quad T_2 (r) + R_2 (r) = 1 \quad \text{instantaneously.} \quad (53)$$

Where  $\omega$  is the wave number of the incoming wave and  $y$  is the *eikonal*.  $T_{1,2}$  and  $R_{1,2}$  are the instantaneous transmission and reflection coefficients [26] respectively. Let us note that the above solution is valid when  $(1/\omega) (d\omega/d\hat{r}) \ll \omega$ .

## 4 Discussion

In this section, we are going to expose the effect of the external parameter  $a$  on the effective potentials by plotting the potentials as a function of radial distance with varying  $a$ . Next, we will study the behavior of the potentials by drawing potentials curves for some different value of frequency  $k$  and charge  $q$  of the spin-1/2 particle. From the potentials Eq. (45), it is very clear that the potentials depend strictly on the external parameter  $a$  and the charge of the particle. Recall that the external parameter  $a$  has been discussed in details [7], indeed  $a$  interpolates between two well-known solutions, namely, the RN and BR solutions. The physical result from this merging is that for a marginally formed RN black hole the black hole property will be lost if the collapse star radius  $r_s$  satisfies  $M^2 + a\sqrt{M^2 - e^2} < r_s < \sqrt{M^2 - e^2}$ , where  $e$  is the black hole charge. For the case of S embedded with BR solution, horizon shrinks as in (4).

To examine the effect of external parameter  $a$  on the potentials we obtain two dimensional plots of Eq. (45). In fig. 1, we exhibit the behaviour of the potentials  $V_{\pm}$  for different values of  $a$ , the case ( $a = 0$ ) is excluded because it lead us to extramal RN case in the metric, where we have chosen the rest mass to be  $\mu_* = 0.12$ , and fixed values of  $k = 0.2, q = 1$  such that  $\mu_* < k$ . It is seen from fig.1 that, potentials have sharp peaks in the physical distance of  $r$ , and when the external parameter  $a$  increases the level of the sharp peaks increases. We

conclude that for large value of  $a$ , a massive charged spin-1/2 particle moving in the physical region faces a high potential barriers whereas for small value of  $a$  encounters low potential barriers. Therefore, as the external parameter increases the kinetic energy of the particle decreases and hardly advances because of the strong retarding potentials.

To examine the behaviour of the potentials for some values of frequencies we obtain figure 2 for various value of  $k$  and fixed value of  $a = 0.4$ . It is seen from fig. 2 that, the general behavior of the potentials are not changed, they still have sharp peaks and behave similar for large distances. Indeed, we notice that at high frequencies the sharp peaks are clear, but for low frequencies the level of sharp peaks decrease. However, Figure 3 shows the nature of the potential changes for different values of the particle charge  $q$ , where the fixed values are ( $a = 0.4$  and  $k = 0.2$ ). It is very clear from fig. 3 that, for small value of  $q$ , the corresponding potential barrier have sharp peaks. It is also noticed that as  $q$  increases the level of sharp peaks decreases. The peaks seem to disappear after a certain value of  $q$ . Therefore we conclude that increasing the charge of the particle reduces the potential barriers and resulting in increasing the kinetic energy of the particle.

Now, the three dimensional plots of the potentials with respect to the external parameter and the radial distance  $r$  is given in figures 4 and 5. Figures 4 and 5 show the effect of the external parameter on the the potentials explicitly for the massive charged spin-1/2 particle, for fixed values of  $\mu_* = 0.12$ ,  $q = 1$  and  $k = 0.2$ . It is seen from figures 4 and 5 that, high potential barriers are observed for large values of external parameter whereas for small value the potential barriers decrease. Again for large distances, potentials levels decrease and asymptote behaviour is manifested. The three dimensional plot of potential with respect to frequency  $k$ , for fixed values of  $a = 0.4$ ,  $q = 1$  and  $\mu_* = 0.12$ , and the radial distance is given in figure 6. We observe from Fig. 6 that, sharp peaks are clear for high frequencies only. Finally, the three dimensional plot of potential with varying  $q$  is given in fig. 7. We can observe from fig. 7 that as the charge of the particle increases the potential levels off.

## 5 Conclusion

In this paper, we have considered the Dirac equation in Schwarzschild black hole immersed in an electromagnetic universe, using NP null tetrad formalism. By employing an axially symmetric ansatz for the Dirac spinor, we managed to separate the equation into radial and angular parts. The angular equations obtained in this geometry are similar to the Schwarzschild case where their solutions are given in terms of standard spherical harmonics. For the radial equations we were able to obtain the radial wave equations with effective potentials. Finally, we studied the behaviour of the potentials by varying the external parameter, charge and the frequency of the spin-1/2 particle, by plotting two and three dimensional plots. We showed that, as the external parameter and the frequency increase, potentials barriers become high and sharp peaks are clear.

However, as the charge of the particle increases potential levels decreases. Our main motivation in the present paper paves the way to study the quasi-normal modes associated to a field of spin-1/2 on the SEBH background. Also the given analytical expressions of the solution could be useful for further study of the thermodynamical properties of the spinor field in same background. For future work, we may generalized our present case by studying the Dirac particles in RN black hole coupled to an external, stationary electromagnetic field.

## References

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Figure 1: Family of potentials graphs  $V_-$  ( solid curves) and  $V_+$  ( dashed curves) for different values of external parameter  $a$ , with  $\mu^* = 0.12, k = 0.2, M = 0.5$  and  $q = 1 = \lambda$ . From the upper to the lower curves the external parameter  $a$  is chosen as  $0.8, 0.6, 0.4, 0.2$ .

Figure 2: Family of potentials graphs  $V_-$  ( solid curves) and  $V_+$  ( dashed curves) for different values of frequency  $k$ , with  $\mu^* = 0.12, a = 0.4, M = 0.5$  and  $q = 1 = \lambda$ . From the upper to the lower curves the  $k$  is chosen as  $1, 0.8, 0.6, 0.4, 0.2$ .

Figure 3: Family of potentials graphs  $V_-$  (solid curves) and  $V_+$  (dashed curves) for different values of charge  $q$ , with  $\mu^* = 0.12, k = 0.2, M = 0.5, a = 0.4$  and  $\lambda = 1$ . From the lower to the upper curves the charge  $q$  is chosen as 1, 0.8, 0.6, 0.4, 0.2.

Figure 4: Three dimensional plot of the potential  $V_-$  for different values of external parameter  $a$ , with  $\mu^* = 0.12, k = 0.12, M = 0.5$  and  $q = 1 = \lambda$ .

Figure 5: Three dimensional plot of the potential  $V_+$  for different values of external parameter  $a$ , with  $\mu^* = 0.12, k = 0.12, M = 0.5$  and  $q = 1 = \lambda$ .

Figure 6: Three dimensional plot of the potential  $V_+$  for different values of frequency  $k$ , with  $\mu^* = 0.12$ ,  $a = 0.4$ ,  $M = 0.5$  and  $q = 1 = \lambda$ .



Figure 7: Three dimensional plot of the potential  $V_-$  for different values of charge  $q$ , with  $\mu^* = 0.12$ ,  $a = 0.4$ ,  $M = 0.5$ ,  $k = 0.2$  and  $\lambda = 1$ .